# A-LEVEL
# MATHEMATICS

Mechanics 4 – MM04
Mark scheme

6360
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**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| –*x* EE | deduct *x* marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| 1 | $\mathbf{r_1 x F_1}=\begin{vmatrix} \boldsymbol{i} & 1 & 3 \\ \boldsymbol{j} & -2 & 1 \\ \boldsymbol{k} & a & 4 \end{vmatrix} = \begin{pmatrix} -8-a \\ 3a-4 \\ 7 \end{pmatrix}$ | **M1** | | Use of **r x F** or **F x r** – at least two components correct in any vector |
| | $\mathbf{r_2 x F_2} = \begin{vmatrix} \boldsymbol{i} & 6 & \frac{1}{2} \\ \boldsymbol{j} & 2 & -1 \\ \boldsymbol{k} & 8 & \frac{1}{2}a \end{vmatrix} = \begin{pmatrix} a+8 \\ 4-3a \\ -7 \end{pmatrix}$ | | | |
| | $\mathbf{r_3 x F_3}=\begin{vmatrix} \boldsymbol{i} & 1 & a \\ \boldsymbol{j} & 0 & 2 \\ \boldsymbol{k} & -1 & -a \end{vmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ | **A(2,1,0)** | | **A1** – Two vector products fully correct<br>**A2** – All three correct |
| | $\mathbf{r_1 x F_1} + \mathbf{r_2 x F_2} + \mathbf{r_3 x F_3} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ | **m1A1F** | | **m1** – Dependent on summing three vector products and the first **M1**<br>**A1F** – their correct sum – following through one slip only |
| | $a$ cancels - hence independent of $a$ | **E1** | | Comment indicating independent of $a$ - **CSO** |
| | **Total** | | **6** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **2(a)** | Volume $= \pi \int_0^a a^2 x \, dx$ | **M1** | | Use of correct formula seen |
| | $= \pi \left[\frac{a^2 x^2}{2}\right]_0^a$ | **A1** | | Correct integration |
| | $= \frac{1}{2}\pi a^4$ | **A1** | 3 | **CSO** - Printed answer |
| **(b)** | $\pi \int_0^a xy^2 \, dx = \pi \int_0^a a^2 x^2 \, dx$ | | | |
| | $= \pi \left[\frac{a^2 x^3}{3}\right]_0^a$ | **M1** | | Correct use of formula - integrating $kx^2$ |
| | $= \frac{1}{3}\pi a^5$ | **A1** | | **CAO** |
| | $\bar{x} = \frac{\frac{1}{3}\pi a^5}{\frac{1}{2}\pi a^4}$ | **M1** | | Their answer for $\pi \int_0^a xy^2 \, dx \div$ part (a) |
| | $= \frac{2a}{3}$ | **A1** | 4 | **CAO** |
| **(c)** | $\tan 30^0 = \frac{distance\ of\ c\ of\ m\ from\ base}{radius\ of\ base}$ | **M1** | | Forming an equation using $\tan 30^0$ or $\tan 60^0$ |
| | $\tan 30^0 = \frac{\frac{a}{3}}{a^{\frac{3}{2}}}$ | **A1** | | Fully correct equation formed – correct expressions for distances seen – accept equivalent, unsimplified expressions |
| | $a^{-\frac{1}{2}} = \sqrt{3}$ | **m1** | | Solving their equation using correct value for $\tan 30^0$ or $\tan 60^0$ - dependent on first **M1** |
| | $a = \frac{1}{3}$ | **A1F** | 4 | Correct solution to **their** equation – must have both **M**s above |
| | **Total** | | **11** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **3** | MI for particle at B = $2(0.3)^2 = 0.18$ | **M1** | | Use of $md^2$ at least once |
| | MI for particle at D = $m(0.4)^2 = 0.16m$ | **A1** | | Correct MI for both B and D |
| | MI for particle at C = $6(0.5)^2 = 1.5$ | **A1** | | Correct MI for particle C<br>All terms can be unsimplified – **M0** if formulas for rods are used |
| | Forming an equation, | | | |
| | $1.68 + 0.16m = 2.24$ | **m1** | | Forming an equation to find $m$ – dependent on first **M1** |
| | Hence $m = \frac{0.56}{0.16} = 3.5$ | **A1** | 5 | Correct value of $m$ obtained - **CAO** |
| | **Total** | | **5** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **4 (a)** | Consider forces horizontally at A, | | | |
| | $T_{AB} = T_{AE} \cos 60^0 = 50 \cos 60^0 = 25$ N | **B1** | | **CAO** |
| | AB is in tension | **E1** | 2 | **CAO** |
| **(b)** | Consider forces vertically at A, | | | |
| | $R_A = T_{AE} \sin 60^0 = 50 \sin 60^0 = 25\sqrt{3}$ | **B1** | 1 | **CAO** – accept any equivalent (eg 43.3) |
| **(c)** | Resolving vertically for whole framework, | | | |
| | $x = R_A + R_C$ | | | |
| | By symmetry, $R_A = R_C$ | **M1** | | Resolving and use of symmetry seen |
| | $x = 2R_A = 50\sqrt{3}$ | **A1F** | 2 | Follow through part b) -<br>$x$ = twice their answer to part b)<br>accept any equivalents (eg 86.6) |
| **(d)** | $T_{ED} = T_{AE}\cos 60^0 + T_{EB}\cos 60^0$ | **M1** | | **M1** Resolving forces horizontally at E (or D) – at least one correct term seen on right hand side |
| | Considering vertical forces at E,<br>Rod EB must be in tension with magnitude 50N | | | |
| | Hence $T_{ED} = 50$N | **A1** | | **CAO** |
| | ED is in compression | **E1** | 3 | **CAO** |
| | **Total** | | **8** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **5(a)** | $\begin{pmatrix} 6-2p \\ p-3 \end{pmatrix}$ | **B1** | 1 | |
| **(b)** | Moments about O | | | |
| | = -1(3) – 5(2) – 5(4) + $p$(3) | **M1** | | Two pairings correct – force x distance |
| | | **A1** | | All pairings correct |
| | | **A1** | | All signs consistent |
| | = 3$p$ -33 or 3($p$-11) | **A1** | 4 | $c$ = 3 (or -3 if directions interchanged) |
| **(c)** | $6-2p=0$ and $p-3=0$ | **M1** | | Setting components/resultant force equal to zero from part a) |
| | Hence $p=3$ (is consistent solution) | **A1** | | Correct and consistent $p$ value obtained |
| | Magnitude = 24 | **B1** | | Correct magnitude of couple stated |
| | Sense is clockwise | **B1** | | Correct sense of couple stated |
| | No linear resultant and non-zero moment therefore is a couple | **E1** | 5 | Fully correct deduction concerning couple |
| **(d)(i)** | **F** = $\begin{pmatrix} -16 \\ 8 \end{pmatrix}$ | **M1** | | **M1** -Substitution of their $p$ value in a) |
| | | **A1** | 2 | **A1** Correct **F** seen - **CAO** |
| **(ii)** | When $p$ = 11, resultant moment about O is 0 hence line goes through origin | **M1** | | Use of zero moment seen or comment made |
| | Line L is $y=-\frac{1}{2}x$ | **A2** | 3 | **CAO** |
| | **Total** | | **15** | |

**Alternative for b)**

$\begin{vmatrix} \boldsymbol{i} & 0 & 1 \\ \boldsymbol{j} & 3 & 2 \\ \boldsymbol{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$ **M1 –** One determinant attempted – at least two components correct

$\begin{vmatrix} \boldsymbol{i} & 2 & -2p \\ \boldsymbol{j} & 0 & -5 \\ \boldsymbol{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix}$ **A1 –** two determinants evaluated correctly

$\begin{vmatrix} \boldsymbol{i} & 3 & 5 \\ \boldsymbol{j} & 4 & p \\ \boldsymbol{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3p-20 \end{pmatrix}$ **A1 –** All three correct

**Total** = $\begin{pmatrix} 0 \\ 0 \\ 3p-33 \end{pmatrix}$

Hence c = 3 **A1 –** correct sum obtained or $c$ = 3 stated from fully correct working

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **6(a)** | $T_1 - 2mg = 2ma$<br>$4mg - T_2 = 4ma$ | **M1** | | Forming two correct equations of motion for particles |
| | $2(T_1 - 2mg) = 4mg - T_2$ | **m1** | | Correct elimination of $a$ (acceleration) |
| | Hence $2T_1 + T_2 = 8mg$ | **A1** | 3 | Fully shown – printed answer - **CSO** |
| **(b)** | Using $C = I\ddot{\theta}$<br>$rT_2 - rT_1 - \frac{1}{2}mgr = 6mr^2\ddot{\theta}$ | **M1A1** | | Correct use of equation of motion for the pulley. **M1** for LHS with inclusion of all turning forces. **A1** Fully correct |
| | Cancelling r gives<br>$T_2 - T_1 - \frac{1}{2}mg = 6mr\ddot{\theta}$ | **A1** | 3 | **CSO** – printed answer |
| **(c)** | From a) adding equations gives<br>$T_1 - T_2 + 2mg = 6ma$ | | | |
| | In b) $T_2 - T_1 - \frac{1}{2}mg = 6ma$ | | | |
| | Hence $T_2 - T_1 - \frac{1}{2}mg = T_1 - T_2 + 2mg$ | **M1A1** | | **M1** Use of elimination to obtain a second equation involving $T_1$ and $T_2$<br>**A1** Correct equation obtained |
| | Gives $T_2 - T_1 = \frac{5mg}{4}$ | | | |
| | Solving with a) $2T_1 + T_2 = 8mg$ | **m1** | | Solving a pair of equations to obtain expressions for $T_1$ and $T_2$. Dependent on first **M1** |
| | Gives $T_1 = \frac{9mg}{4}$ and $T_2 = \frac{7mg}{2}$ | **A1** | 4 | Both $T_1$ and $T_2$ correct - **CSO** |
| **(d)** | | | | |
| | Using $T_2 - T_1 - \frac{1}{2}mg = 6mr\ddot{\theta}$ | **M1** | | Substituting their $T_2$ and $T_1$ and rearranging to obtain expression for $\ddot{\theta}$ |
| | $\ddot{\theta} = \frac{g}{8r}$ | **A1F** | 2 | Must involve $g$ and $r$ only |
| | **Total** | | **12** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **7(a)(i)** | $m = \pi a^2 h\rho$ or $\rho = \frac{m}{\pi a^2 h}$ | **B1** | | Correct equation connecting $m$ and $\rho$ seen |
| | Mass of thin disc = $\pi a^2 dx\rho$<br>MI of thin disc = $\frac{1}{2}(\pi a^2 dx\rho)a^2$ | **M1** | | Formulating MI of elemental disc |
| | MI of cylinder<br>$= \int_0^h \frac{1}{2}\pi a^4 \rho dx = \left[\frac{1}{2}\pi a^4 x\rho\right]_0^h$ | **A1** | | Integrating correctly |
| | $= \frac{1}{2}\pi a^4 \frac{m}{\pi a^2 h} h$ | **m1** | | Substituting correct limits and $p$ – dependent on first **M1** |
| | $= \frac{1}{2}ma^2$ | **A1** | 5 | Obtaining printed answer - **CSO** |
| | **Alternative for (a)(i)**<br>$m = \pi a^2 h\rho$ or $\rho = \frac{m}{\pi a^2 h}$ | **(B1)** | | Correct equation connecting $m$ and $\rho$ seen |
| | MI of cylindrical shell = $2\pi rh\rho r^2 dr$ | **(M1)** | | Formulating MI of elemental shell |
| | MI of cylinder<br>$= \int_0^a 2\pi rh\rho r^2 dr = \left[\frac{2\pi r^4 h\rho}{4}\right]_0^a$ | **(A1)** | | Integrating correctly |
| | $= \frac{2}{4}\pi a^4 \frac{m}{\pi a^2 h} h$ | **(m1)** | | Substituting correct limits and $p$ – dependent on first **M1** |
| | $= \frac{1}{2}ma^2$ | **(A1)** | (5) | Obtaining printed answer - **CSO** |
| **(b)** | Given cylinder MI = $\frac{1}{2}(2m)(3a)^2$ | **B1** | | Correct adjustment of formula in part a) - can be unsimplified |
| | Using parallel axis theorem<br>$I_{AXIS} = \frac{1}{2}(2m)(3a)^2 + 2m(3a)^2$ | **M1A1** | | **M1** Correct structure of parallel axis theorem<br>**A1** Distance and masses correct |
| | $= 27ma^2$ | **A1** | 4 | **CAO** |
| **(c)(i)** | Forming an energy equation<br>$\frac{1}{2}(27ma^2)\dot{\theta}^2 = (2mg)(3a\sin\theta)$ | **M1**<br>**A1F**<br>**A1** | | **Follow through part b) throughout**<br>Forming an equation using KE gain = PE loss<br>Their correct kinetic energy used (LHS)<br>Correct potential energy used (RHS) |
| | Hence $\dot{\theta}^2 = \frac{4g\sin\theta}{9a}$ | **A1F** | | Rearranging to get $\dot{\theta}^2$ - can be unsimplified |
| | Differentiating gives $2\dot{\theta}\ddot{\theta} = \frac{4g\cos\theta}{9a}\dot{\theta}$ | **M1** | | Differentiating correctly |
| | Hence $\ddot{\theta} = \frac{2g\cos\theta}{9a}$ | **A1F** | 6 | Rearranging to get $\ddot{\theta}$ - can be unsimplified |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **c(ii)** | Using F= *ma* tangentially | | | |
| | $2mg\cos\theta - Y = (2m)(3a)\ddot{\theta}$ | **M1A1** | | **M1** Use of F = *ma* - one side correct<br>**A1** Both sides correct |
| | $Y = 2mg\cos\theta - (2m)(3a)(\frac{2g\cos\theta}{9a})$<br>$Y = \frac{2mg\cos\theta}{3}$ | **A1F** | 3 | Can be unsimplified – follow through substitution of their $\ddot{\theta}$ from c)i) |
| **(c)(i)** | **Alternative for part (c)(i) – final two marks** | | | **Follow through part b) throughout** |
| | $I\ddot{\theta} = (2mg)(3a\cos\theta)$ | **(M1)** | | Correct use of $C = I\ddot{\theta}$ |
| | Hence $\ddot{\theta} = \frac{2g\cos\theta}{9a}$ | **(A1F)** | | Rearranging to get $\ddot{\theta}$ - can be unsimplified |
| **(c)(ii)** | **Alternative to part (c)(ii)** | | | |
| | Take moments about axis of cylinder gives | | | |
| | $(3a)Y = \frac{1}{2}(2m)(3a)^2\ddot{\theta}$ | **(M1A1)** | | Use of moments – one side correct for **M1**<br>Both sides correct for **A1** |
| | Hence $Y = \frac{2mg\cos\theta}{3}$ | **(A1F)** | (3) | Can be unsimplified – follow through substitution of their $\ddot{\theta}$ from c)i) |
| | **Total** | | **18** | |
| | **TOTAL** | | **75** | |